Why glasses do not flow ?

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Glasses are liquids that do not flow. They look like liquids (atoms are in random, dense configurations), but behave like solids. More precisely, the time they need to flow rises extremely rapidly when the temperature is reduced, and soon exceed the age of cathedrals (or of the universe, for that matter). Spin-glasses are, similarly, disordered magnets that remain frozen in random spin configurations, with no apparent order. Both type of systems reveal interesting properties: they *age* (their properties depend on the time they spent in the glassy state), and can memorize their history and *simultaneously* rejuvenate when temperature is changed. A very important experimental observation is that the viscosity of glasses (a dynamical property) seems highly correlated to its entropy, i.e., the number of microscopic configurations in which the glass can get stuck. The smaller this number, the larger the viscosity. The striking observation that makes the 'problem of glasses' interesting is that very many, totally different materials, exhibit the same properties, pointing to the existence of a somewhat universal mechanism: glassy dynamics is physics more than chemistry.

Our understanding of these phenomena, in particular of the very rapid slowing down of the dynamics with temperature, is still fragmentary. Early ideas follow two apparently unrelated paths. One line of thought pictures the glass as a material point lost in a random, complex landscape, with valleys and hills of different heights and depths. Each valley corresponds to a quasi-equilibrium, metastable configuration of the system. The 'point' representing the whole system slowly visits the different valleys, hindered in its exploration by higher and higher barriers. This leads to aging, because the typical time needed to exit a valley is roughly the time needed to get there in the first place.

Another old idea is that of "cooperatively rearranging regions" of increasing length. The dynamics becomes sluggish because larger and larger regions of the material have to move simultaneously to allow for a substantial motion of individual particles. Although this idea seems most reasonable, its reality has remained elusive until recently: a consistent definition of this growing length, its experimental measurement and its calculation within a theoretical model (even highly simplified) are subjects of topical activity. Interestingly, both the above ideas (landscape and cooperative length) are also relevant for the description of other "jammed" systems, such as dense granular assemblies that flow in a very jerky way.

The first quantitative theory of the glass state, starting from interacting atoms and making its way up to compute the viscosity of the liquid as a function of density and temperature is the so-called Mode Coupling Theory (MCT), introduced in this context in the mid-eighties. This theory makes a number of quantitative predictions that can be compared to experiments, some of which in remarkable agreement with observations. However, MCT predicts complete freezing of the liquid at a temperature much above the experimental value. Furthermore, there is no sign, in this theory, of the existence of a growing cooperative length scale that would explain why the liquid becomes a glass. This is because MCT is in fact a "mean-field" theory, where all spatial aspects of the problem are discarded. Mean-field approximations are well known, and give a simplified useful picture of how collective phenomena come about in condensed matter physics (superconductivity, magnetism, etc. and in fact in other fields too, such as economics). However, these mean-field theories usually fail to describe these collective phenomena quantitatively. In the present case, MCT is in fact tantamount to the landscape picture alluded to above. Below a certain temperature, the system gets trapped into one of the (exponentially many) blocked configurations; within MCT, however, the barriers between these blocked states is infinite.

In reality, one expects (theoretically) two crucial modifications to the MCT picture: 1) the freezing transition should be associated to the divergence of a cooperative length scale. Such a length scale was indeed very recently obtained within an extended formulation of MCT; 2) below the transition, the system should break up in 'droplets' of frozen configurations with some mismatch energy between the droplets. The size of these droplets results from a competition between the number of possible configurations (i.e. the configurational entropy) and the mismatch energy; this size gives the dynamical cooperative length and indeed increases when the temperature decreases (because the configurational entropy decreases and even seems to go to zero at the so-called Kauzmann ideal glass temperature).

The relevance of this beautiful scenario, where the liquid slows down because of the emergence of a very large number of metastable states that momentarily trap the system, with larger and larger frozen regions as the system is allowed to visit deeper and deeper energies (i.e.

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more and more jammed states), is however still quite controversial. No realistic model where this scenario can be proven mathematically yet exists. A competing, somewhat simpler scenario also exists, based on the idea of 'free volume', i.e. some sort of vacancies or voids that trigger and catalyze the dynamics of the atoms and allow them to move only in they are close by. As the temperature is reduced, or the density increased, the density of these voids decreases; the cooperative length is in this case simply the typical distance between the voids. In this scenario however, the empirical connection between the dynamics and the configurational entropy is lost.

The same issues exist also for spin-glasses. The mean-field solution in this case reveals an even richer and more complex landscape structure, with valleys with valleys with valleys, in a hierarchical (fractal) fashion. Although this fractal picture is very helpful to account for the memory and rejuvenation effects in these systems, the way to reconcile mean-field with real spin-glasses where the dynamics again becomes slow because of the growth of some cooperative length scale is again far from settled. A remarkable effect predicted in spin-glasses is their extreme fragility to tiny temperature changes, that can induced large rearrangements in the configurations. Such a fragility was also discussed in other contexts (pinned vortex lines, dislocations, domain walls; but also force chains in granular materials). The extent to which this 'temperature chaos' effect also applies to regular glasses and allow one to understand rejuvenation in these materials is an open problem.

So, the outstanding questions that remain before we can say we understand glasses are the following:

- How relevant are mean-field ideas/models for real glasses/spin-glasses? Can one find an exactly soluble, non mean-field model of glasses that would shed some light on the physics of real materials? Can a frozen, non crystalline, glass phase ever exist in finite dimensions or is it always killed by 'hopping processes' out of deep wells (absent from MCT, where barriers are infinite)?

- What is the geometry of moving objects in glasses (strings, clusters)? Is there really, experimentally, a detectable growing dynamical length scale in glassy systems (including jamming granular materials, soft glassy materials, spin glasses)? How large can this length really grow (probably not more than 10-20)? Is cooperativity trivial (non thermodynamical) as in mobility defect theories or non trivial (related to an exponential degeneracy of metastable states) as in mean-field models?

- Are structural glasses also, in some sense, fragile to temperature changes?

From a wider perspective, one can ask how much glassy systems, with their profusion of quasi-equilibrium states and complex dynamics, can be used as metaphors in other contexts. Combinatorial optimization is a already a well studied one. Applications in economics/finance/game theory, where equilibrium is often assumed but may never be reached, is a fascinating prospect.